Equivalent Complex Permeability for Soft Magnetic Composites: Application to Transformer

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In this contribution, we conduct homogenization on a transformer made of Soft Magnetic Composites (SMC). High concentration periodic SMC is homogenized with an effective complex permeability, which contains the magnetic behaviour in the real part and the loss characteristics in the imaginary part. It has been found that the higher the concentration of the inclusion, the more accurate the prediction of the magnetic behaviour and eddy current losses of the composites. The equivalent homogeneous transformer using this complex permeability is shown to describe with a satisfying agreement the magnetic field and eddy current loss distribution of the heterogeneous one.

***Index Terms*—Eddy currents, Effective property, Homogenization, Material modeling, SMC, Transformer**

I. Introduction

Soft magnetic composites (SMC) are a new type of material created by powder metallurgy technology. They consist in a collection of ferromagnetic inclusions embedded in a dielectric insulating polymer matrix. Because of this microstructure, SMC possess the characteristics of low level of Eddy Current (EC) losses observed in these materials when they are subjected to electromagnetic loadings. A model of effective complex permeability to represent both the magnetic behaviour (real component) and EC loss density (imaginary component) for periodic SMC with circular or spherical inclusions has been recently developed [1]. Similarly, complex permeability models have been used to homogenize the multi-turn coil of transformers [2]. These models are capable of turning the quasimagnetostatics problems into statics ones and estimating plausibly the EC losses.

In this abstract, EC loss density formula is derived for periodic high concentration SMC with square shaped inclusions and the corresponding complex permeability is deduced. The method is applied to describe a basic transformer structure. Comparison between the equivalent homogeneous transformer and the heterogeneous one is carried out by Finite Element Methods (FEM).

II. Homogenization

A schematic transformer is depicted in Fig. 1. It is made of high concentration periodic SMC with square inclusions. The copper coils are wound to provide magnetic field input to the transformer.

In the following, the EC loss density equation is first deduced for this periodic microstructure of SMC and the corresponding complex permeability is given. The complex permeability is then used to model the transformer performance.

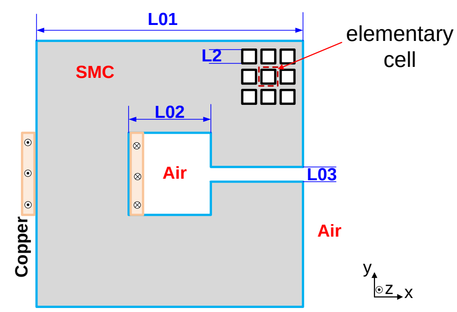

Fig. 1. Sketch of a schematic transformer made of SMC with square inclusions. The domain confined by red dashed lines forms an elementary cell of periodic pattern.

A. EC loss density definition

According to Faraday's law, a time-variation of the magnetic field generates an electric field $\mathbf{E}$. In a conductor Ω of conductivity σ, eddy current $\mathbf{J}_{eddy} = \sigma\mathbf{E}$ arises, which further results in EC losses in the material. The EC loss density $\mathcal{U}$ is defined as the Joule losses dissipated per unit volume during a wave period:

$$\mathcal{U} = \frac{\langle \sigma \mathbf{E}^2 \rangle}{2\,f} \tag{1}$$

where f is the frequency and the operator $\langle\cdot\rangle$ denotes a volume average over the domain Ω by $\langle\cdot\rangle = \frac{1}{V}\int_V \cdot\, \mathrm{d}V$, where V is the volume of Ω.

According to this definition, EC loss density formulas for high concentration SMC are derived, considering a harmonic magnetic field with low frequency f such that the skin effect can be neglected.

B. Complex permeability

SMC have 3D microstructures, but this study will consider a 2D transformer made of 2D SMC (with infinitely long grains along the third direction) in order to evaluate the performance of the homogenization model. The study of a 3D transformer made of 3D SMC would require to solve a more complex numerical system.

The 2D SMC problem can then be separated into two different cases depending on the direction of the magnetic field excitation. The first case considers the magnetic field perpendicular to the cell. The second case considers the magnetic field in-plane. For the first case, if the frequency is low, the magnetic field in the cells can be regarded as uniform. Analytical or semi-analytical models have been proposed to calculate EC losses for laminated steel [3] and for SMC [4].

For the in-plane case, the magnetic field in the inclusion is considered uniform and noted $\mathbf{H}_2 = [H_{2x}, H_{2y}]^t$, where t is the transpose operator. Neglecting the induced magnetic field, the electric field in the inclusion can be solved from Maxwell-Faraday equation as:

$$\mathbf{E}_2(x, y) = -j2\pi f \mu_2 (H_{2x}\, y - H_{2y}\, x)\vec{u}_z \qquad (2)$$

where μ_2 is the magnetic permeability of the inclusion, $\vec{u}_z$ is the unit vector for the chosen coordinate system.

Substituting $\mathbf{E}_2(x, y)$ into the EC density definition (1) and considering that EC losses occur only in the inclusion, the formula of EC loss density of SMC becomes,

$$\mathcal{U} = \frac{\pi^2}{6}\xi_2\, f\, L_2^2\, \sigma_2\, \mu_2^2\, \mathbf{H}_2^2 \qquad (3)$$

where L_2 is the size of the inclusion, and ξ_2 denotes the filling factor (volume fraction) of the inclusions.

The magnetic field $\mathbf{H}_2$ in the inclusion is linked to the macroscopic magnetic field $\overline{\mathbf{H}}$ by [5]:

$$\mathbf{H}_2 = \frac{\tilde{\mu}^r - \mu_1}{\xi_2(\mu_2 - \mu_1)}\overline{\mathbf{H}} \qquad (4)$$

where μ_1 indicates the magnetic permeability of the matrix and $\tilde{\mu}^r$ is the effective magnetic permeability, which can be retrieved satisfactorily by Maxwell-Garnett (MG) estimate [6]:

$$\tilde{\mu}^r = \frac{(\mu_2 + \mu_1) + (\mu_2 - \mu_1)\,\xi_2}{(\mu_2 + \mu_1) - (\mu_2 - \mu_1)\,\xi_2} \qquad (5)$$

According to [1], the imaginary part of the complex permeability is retrieved by losses equivalence $\mathcal{U} = \pi\tilde{\mu}^i\overline{\mathbf{H}}^2$, so that

$$\tilde{\mu}^i = \frac{2}{3}\pi\xi_2\, f\, L_2^2\, \sigma_2 \left(\frac{\mu_1\mu_2}{(\mu_2 + \mu_1) - (\mu_2 - \mu_1)\,\xi_2}\right)^2 \qquad (6)$$

And the complex permeability has the form,

$$\tilde{\mu} = \tilde{\mu}^r - j\tilde{\mu}^i. \qquad (7)$$

The complex permeability of 3D SMC of cubic inclusions placed in cubic lattice form can be similarly obtained, which will be explored in the full paper.

III. Results and conclusion

Magnetic behaviour and loss prediction from the complex permeability model are first examined for high concentration periodic SMC by comparing with the FEM results.

In the calculations, Iron is selected as the inclusion material ($\sigma_2 = 1.12 \times 10^7\,\mathrm{S/m}, \mu_2 = 4000\,\mu_0$), Epoxy as the matrix ($\sigma_1 \approx 0, \mu_1 = \mu_0$), average flux density $B_0 = 1$ T, frequency $f = 100$ Hz. The cell size is fixed at $50\,\mu\mathrm{m}$.

For high filling factor SMC ($> 95\%$), the effective magnetic permeability predicted by the complex permeability model (MG estimate) brings discrepancy less than 1% on an elementary cell when compared to FEM results. Similarly, the complex permeability provides reliable EC losses (errors also less than 1% by comparing with FEM results). For both the magnetic behavior and losses estimates, the higher the filling factor of the inclusions, the smaller the discrepancy.

A heterogeneous transformer made of SMC is modeled; first by using a full description of the heterogeneity, and then using the equivalent complex permeability. By examining the magnetic field in the air gap, it can be concluded that the homogenized transformer has the same magnetic behaviour (variation less than 2%) as the heterogeneous one. The EC losses discrepancies between the heterogeneous and homogeneous transformer are plotted in Fig. 2.

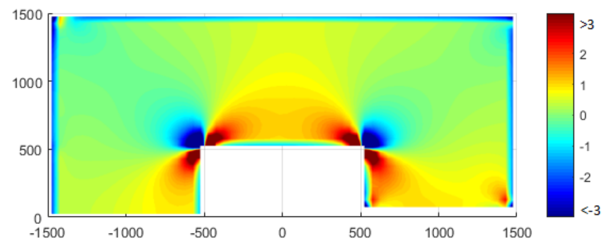

Fig. 2. EC losses discrepancies (%) between heterogeneous and homogeneous transformer.

It can be observed that the overall error on the EC losses is small, less than 3% except for localized areas at the inner corners of the transformer. It is then concluded that the homogenization method can provide EC losses distribution with very satisfying accuracy. Of course local fluctuations are smoothed by the approach.

References

[1] X. Ren, R. Corcolle, L. Daniel, "A Homogenization Technique to Calculate Eddy Current Losses in Soft Magnetic Composites Using a Complex Magnetic Permeability," *IEEE Trans. Magn.*, Vol.52, No. 12, Dec. 2016.

[2] H. Igarashi, "Semi-Analytical Approach for Finite Element Analysis of Multi-turn Coil Considering Skin and Proximity Effects," *IEEE Trans. Magn.*, Vol.53, No. 1, Jan. 2017.

[3] S. K. Mukerji, M. George, M. B. Ramamurthy, and K. Asaduzzaman,, "Eddy currents in solid rectangular cores," *Prog. Electromagn. Res. B*, vol. 7, pp. 117–131, 2008.

[4] O. de la Barrière, M. LoBue, and F. Mazaleyrat, "Semianalytical and analytical formulas for the classical loss in granular materials with rectangular and elliptical grain shapes," *IEEE Trans. Magn.*, vol. 50, pp. 1–8, Oct 2014.

[5] R. Corcolle, L. Daniel, and F. Bouillault, "Intraphase fluctuations in heterogeneous magnetic materials," *J. Appl. Phys.*, vol. 105, no. 12, 2009.

[6] J. C. Maxwell-Garnett, "Colours in metal glasses and in metallic films," *Philos. Trans. Roy. Soc. London A*, vol. 203, no. 359-371, pp. 385–420, 1904.